Example: Regular Pumping Lemma$_{JP}$

The following is a walk-through of the JFLAP Regular Pumping Lemma Game for the lemma

$$L = \{ww^R : w \in (a, b)^*\}$$

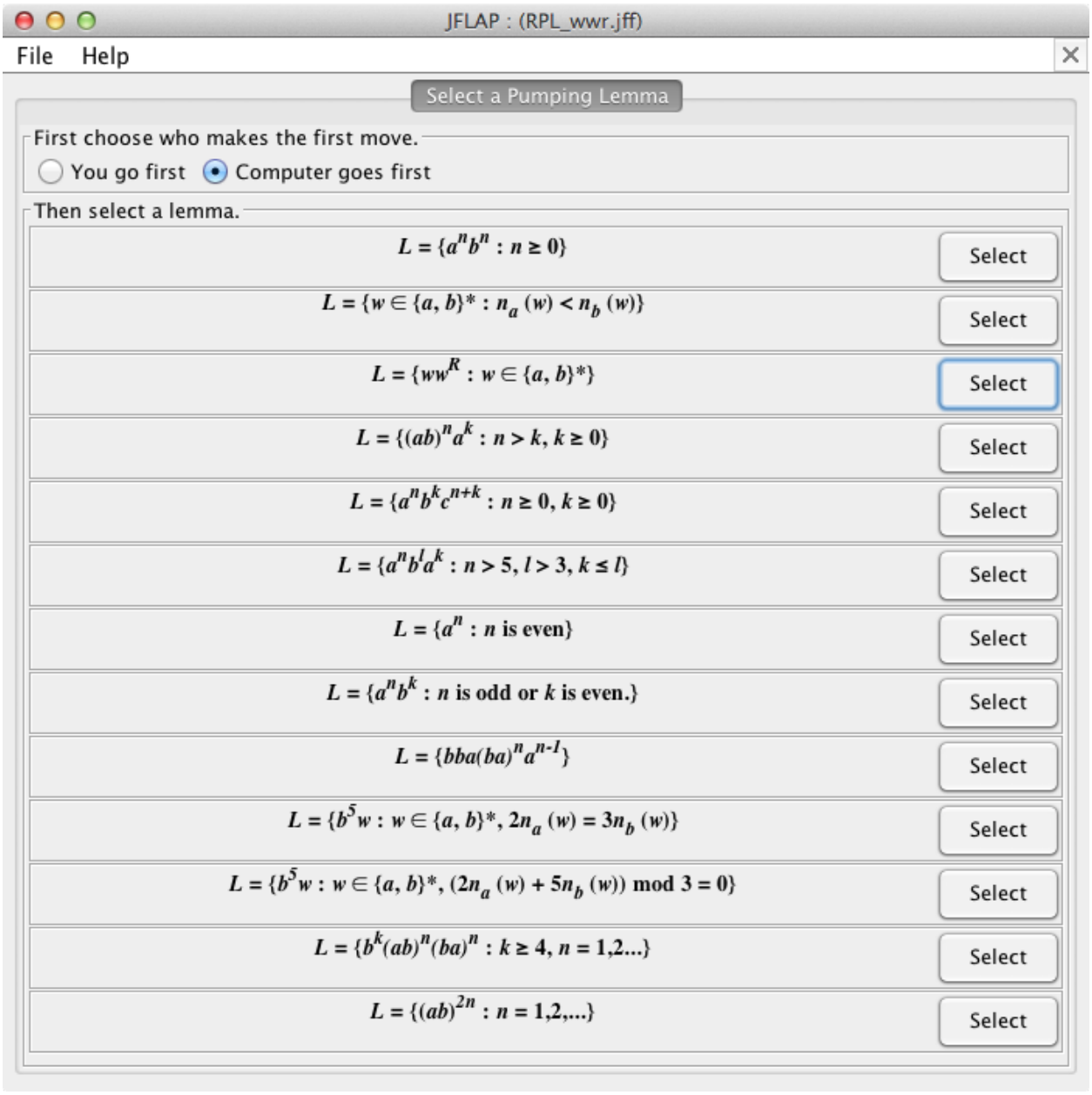
Recall that if L is a regular language then there exists an integer m > 0 such that any $w \in L$ with $|w| \geq m$ can be decomposed as the concatenation $w = xyz$, with $|xy| \leq m$, $|y| \geq 1$, and $xy^iz \in L$ for all $i \geq 0$.

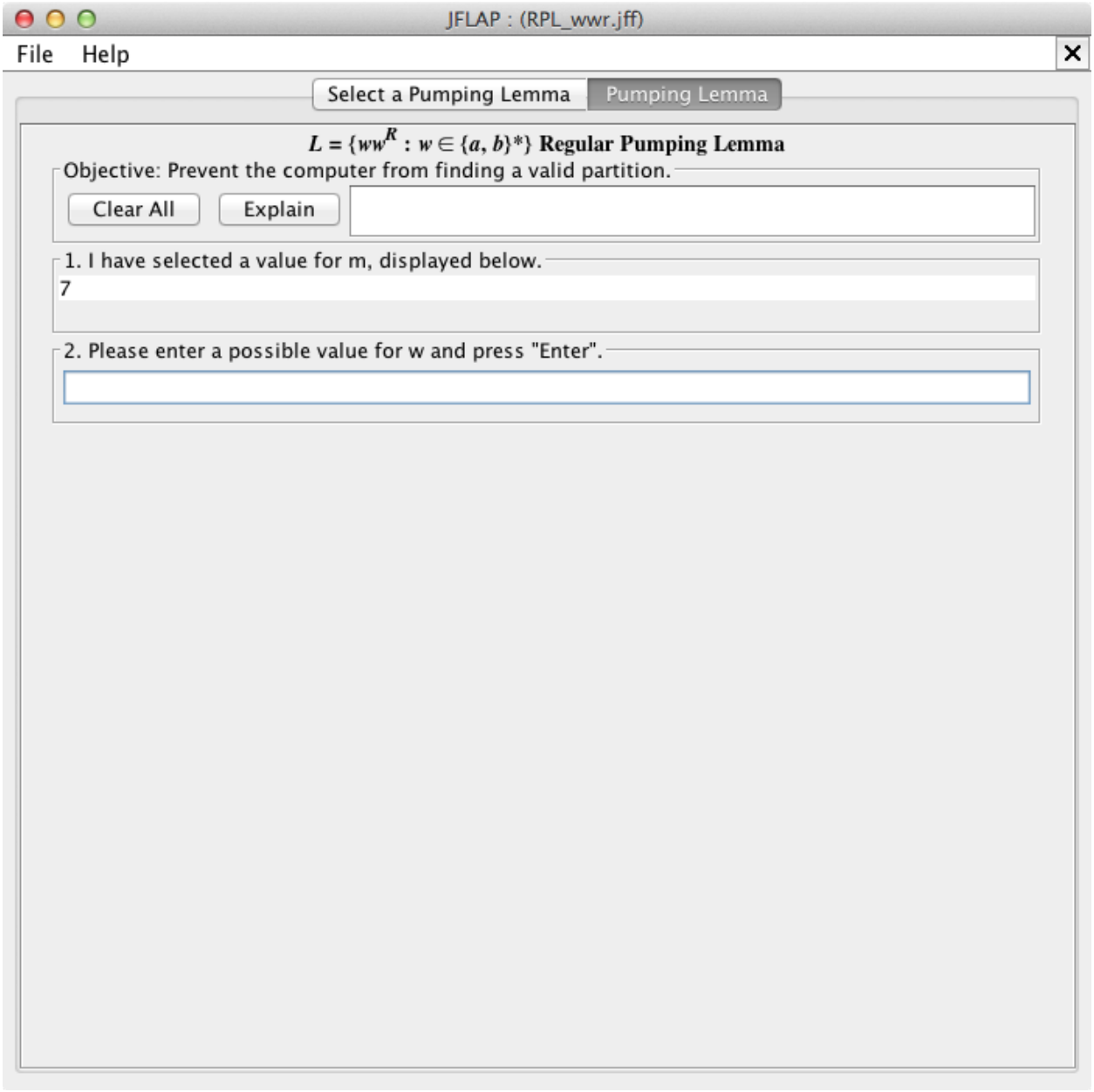
Consider what characteristics of a string are necessary for successful decomposition into three concatenated components, the middle of which can be eliminated or repeated.

Walkthrough (see `RPL_wwr.jff`)

1. Open JFLAP and select *Regular Pumping Lemma*

2. Select “Computer goes first” and choose *Select* to the right of “L = {ww^R : w □ (a, b)*}”



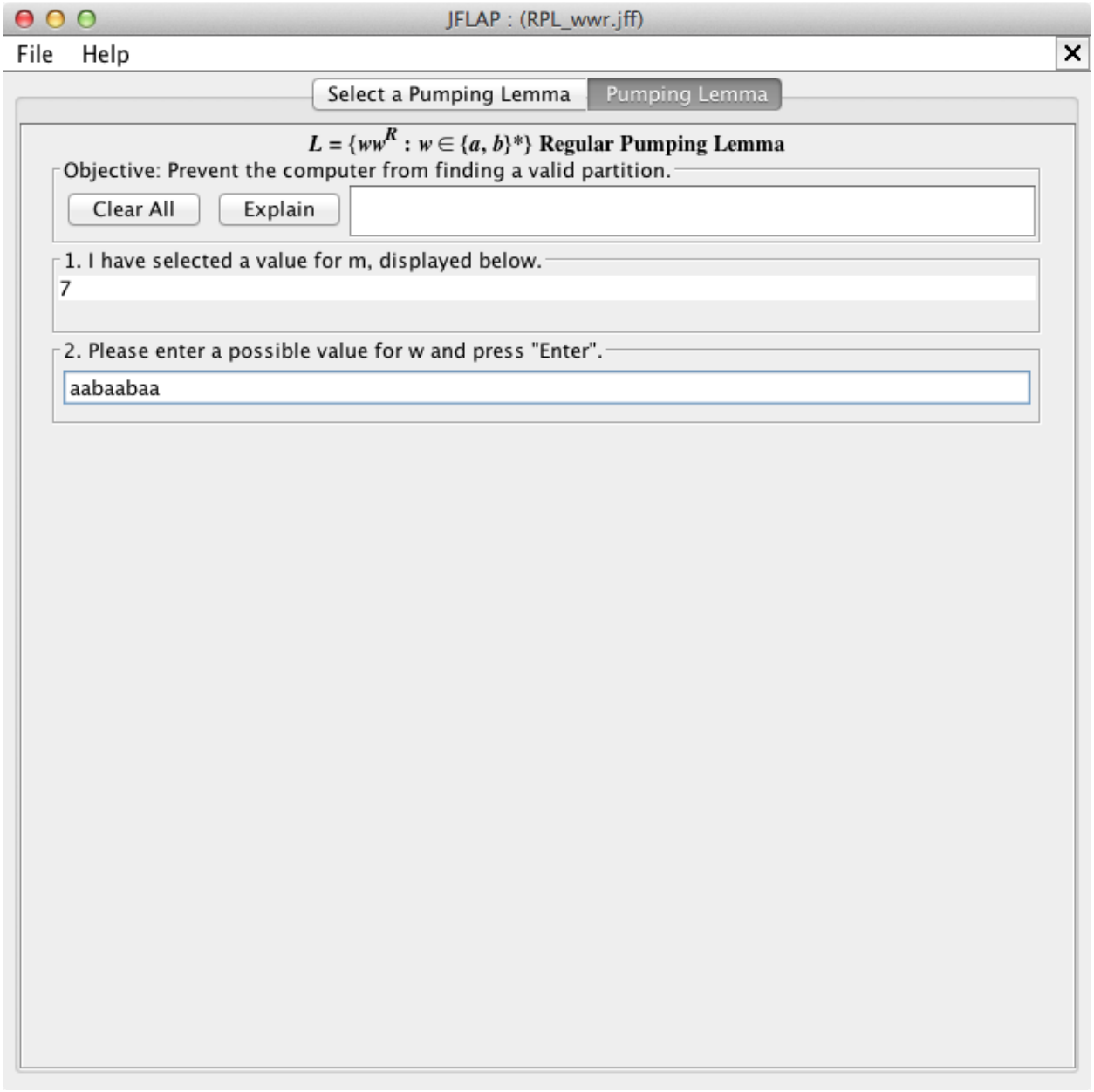


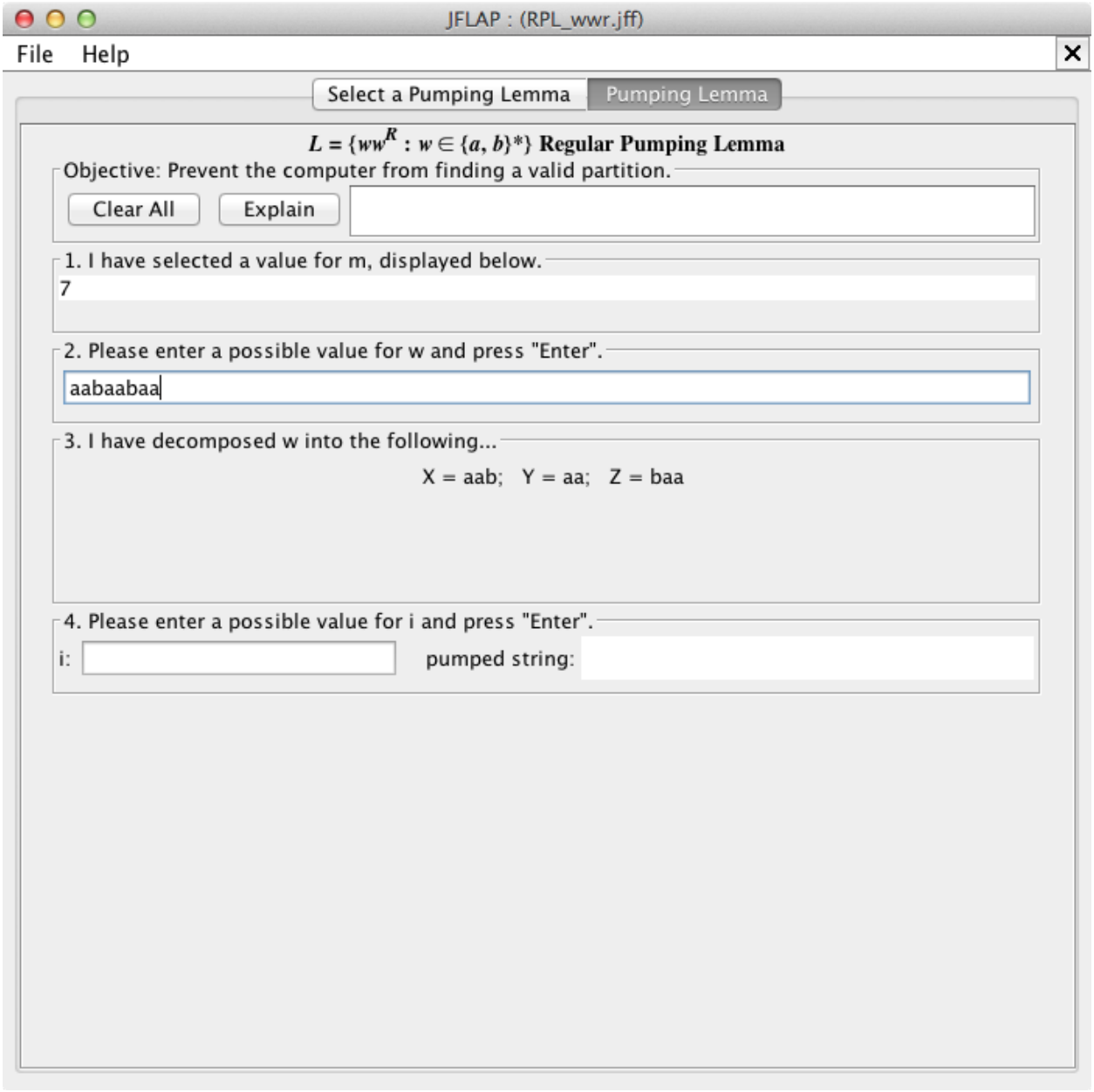
Notice that the computer has selected the value 7 for m.

3. Consider strings of length 7 or greater in language L.

The string ww^R is the concatenation of two strings, w and w^R, and that |w| = $|w^R|$. Therefore all strings in L must have even length.

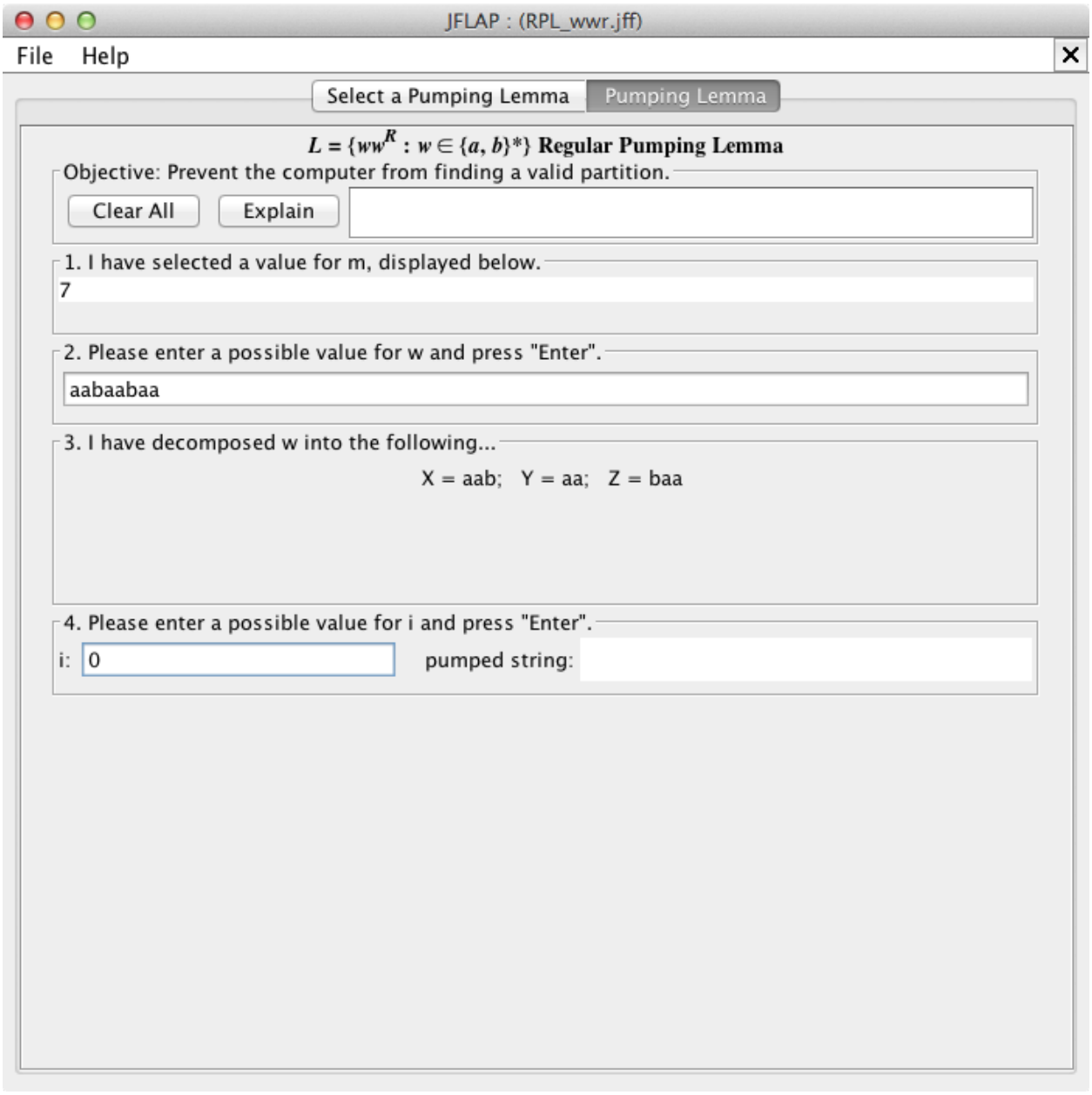
Enter a candidate value for w of length 8, such as aabaabaa.

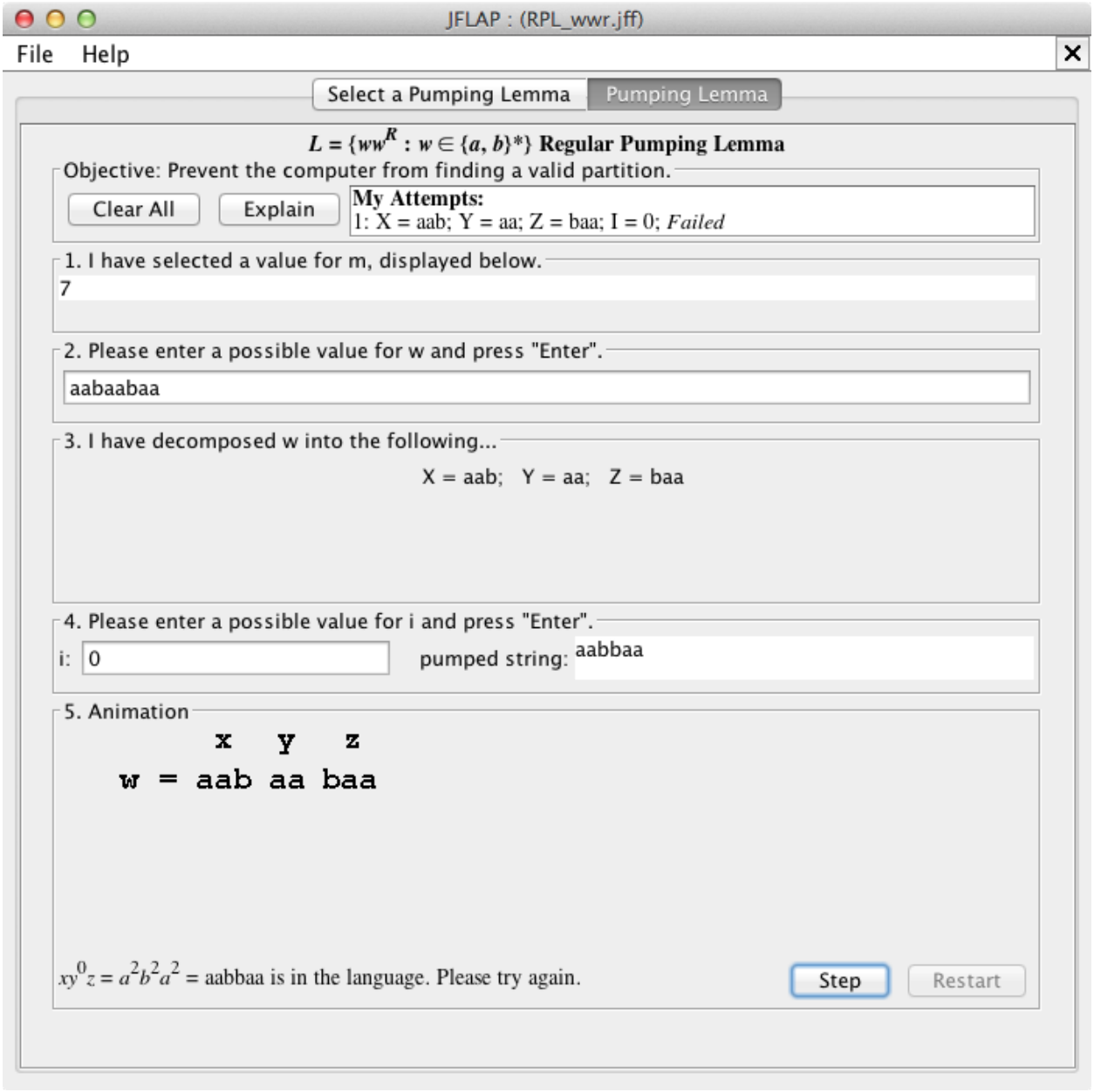

JFLAP : (RPL_wwr.jff)
File
Help
Select a Pumping Lemma
Pumping Lemma
$L = \{ww^R : w \in \{a, b\}^*\}$ Regular Pumping Lemma
Objective: Prevent the computer from finding a valid partition.
Clear All
Explain
1. I have selected a value for m, displayed below.
7
2. Please enter a possible value for w and press "Enter".
aabaabaa



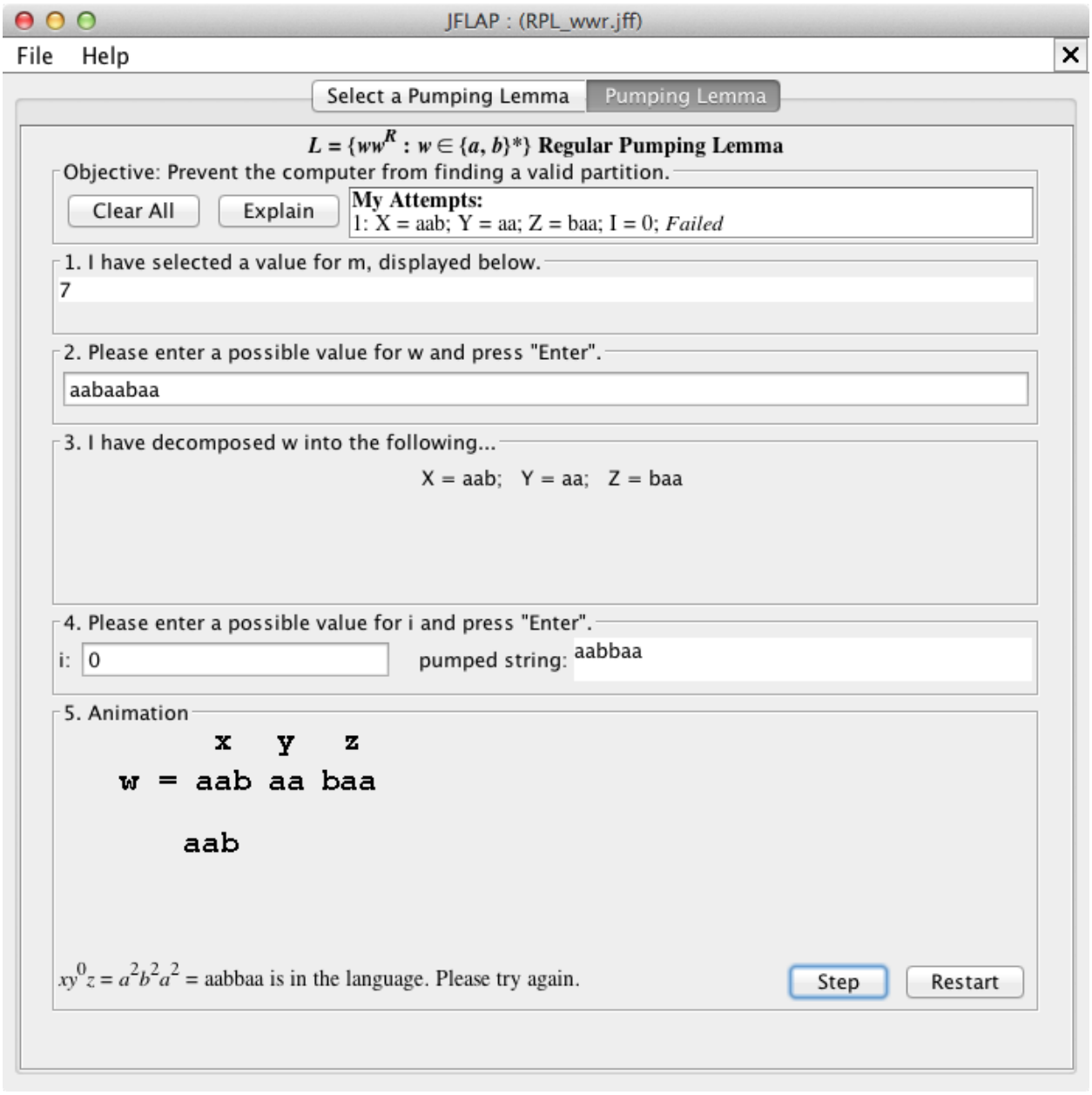
The decomposition chosen by the computer appears to be one that fits all of the requirements of the pumping lemma.

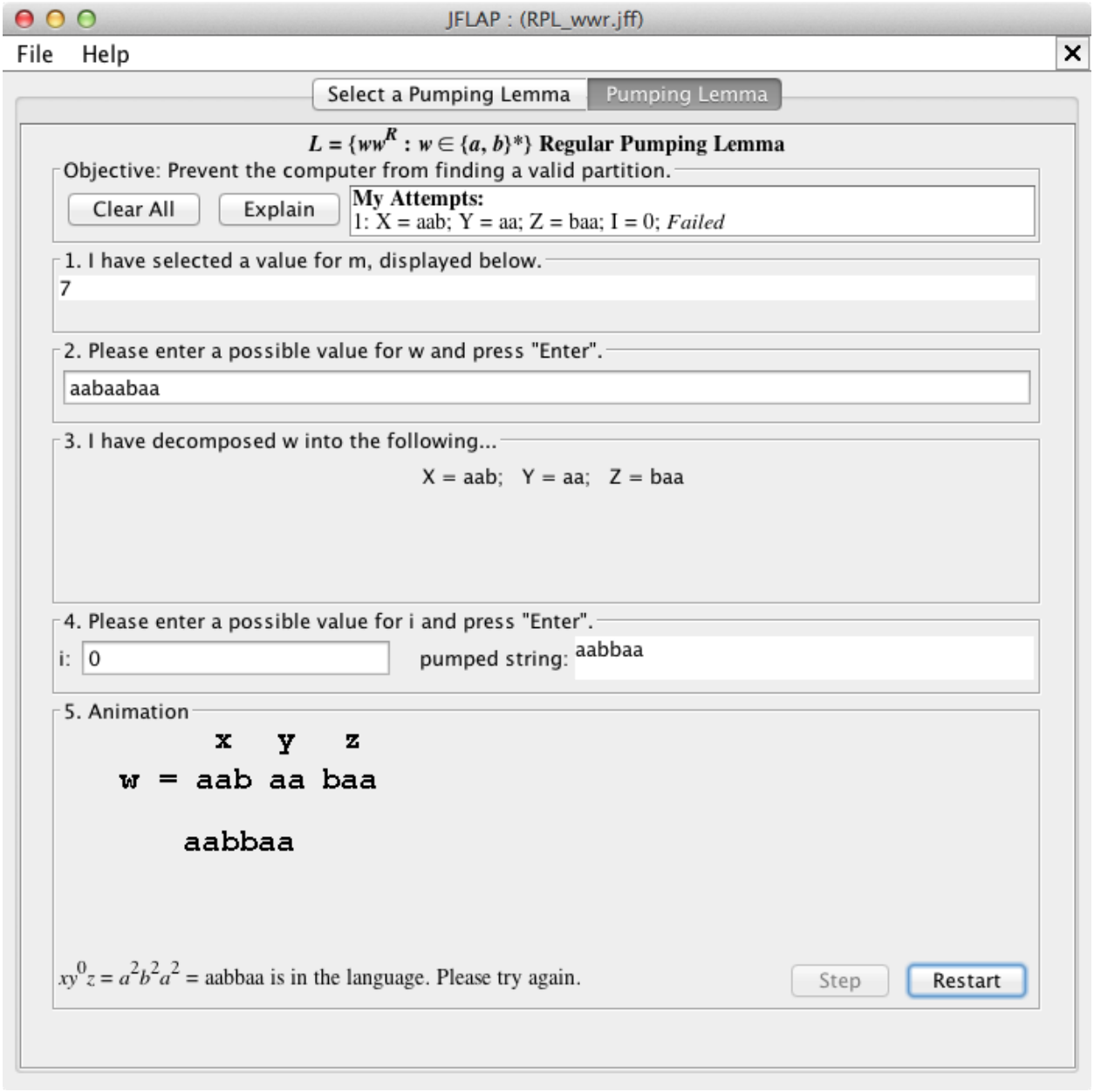
4. Enter any value for i, such as 0, to see the pumped string.



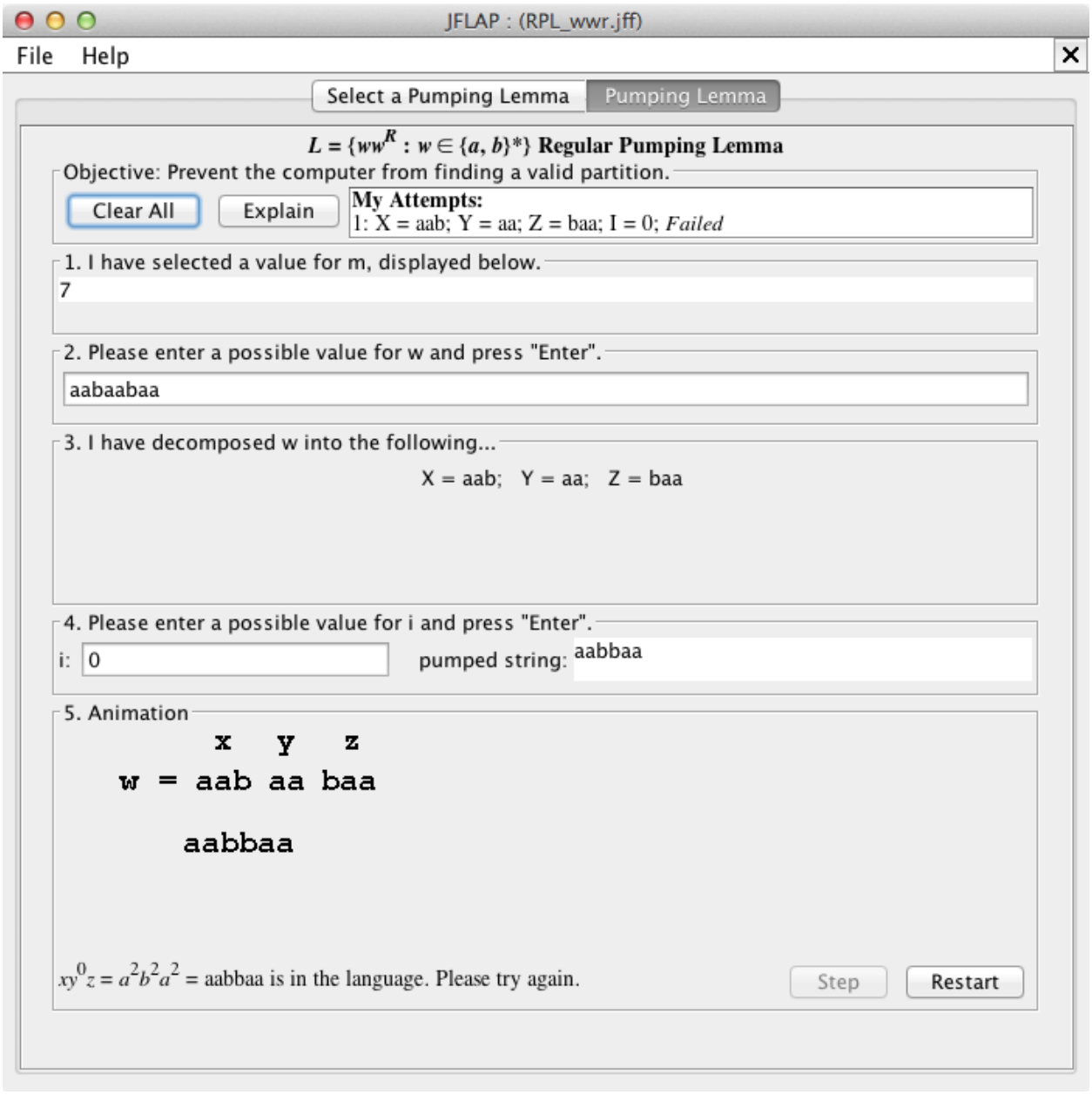

JFLAP : (RPL_wwr.jff)
File Help
Select a Pumping Lemma
Pumping Lemma
$L = \{ww^R : w \in \{a, b\}^*\}$ Regular Pumping Lemma
Objective: Prevent the computer from finding a valid partition.
Clear All
Explain
My Attempts:
1: X = aab; Y = aa; Z = baa; I = 0; Failed
1. I have selected a value for m, displayed below.
7
2. Please enter a possible value for w and press "Enter".
aabaabaa
3. I have decomposed w into the following...
X = aab; Y = aa; Z = baa
4. Please enter a possible value for i and press "Enter".
i: 0
pumped string: aabbaa
5. Animation
x y z
w = aab aa baa
$xy^0z = a^2b^2a^2$ = aabbaa is in the language. Please try again.
Step
Restart

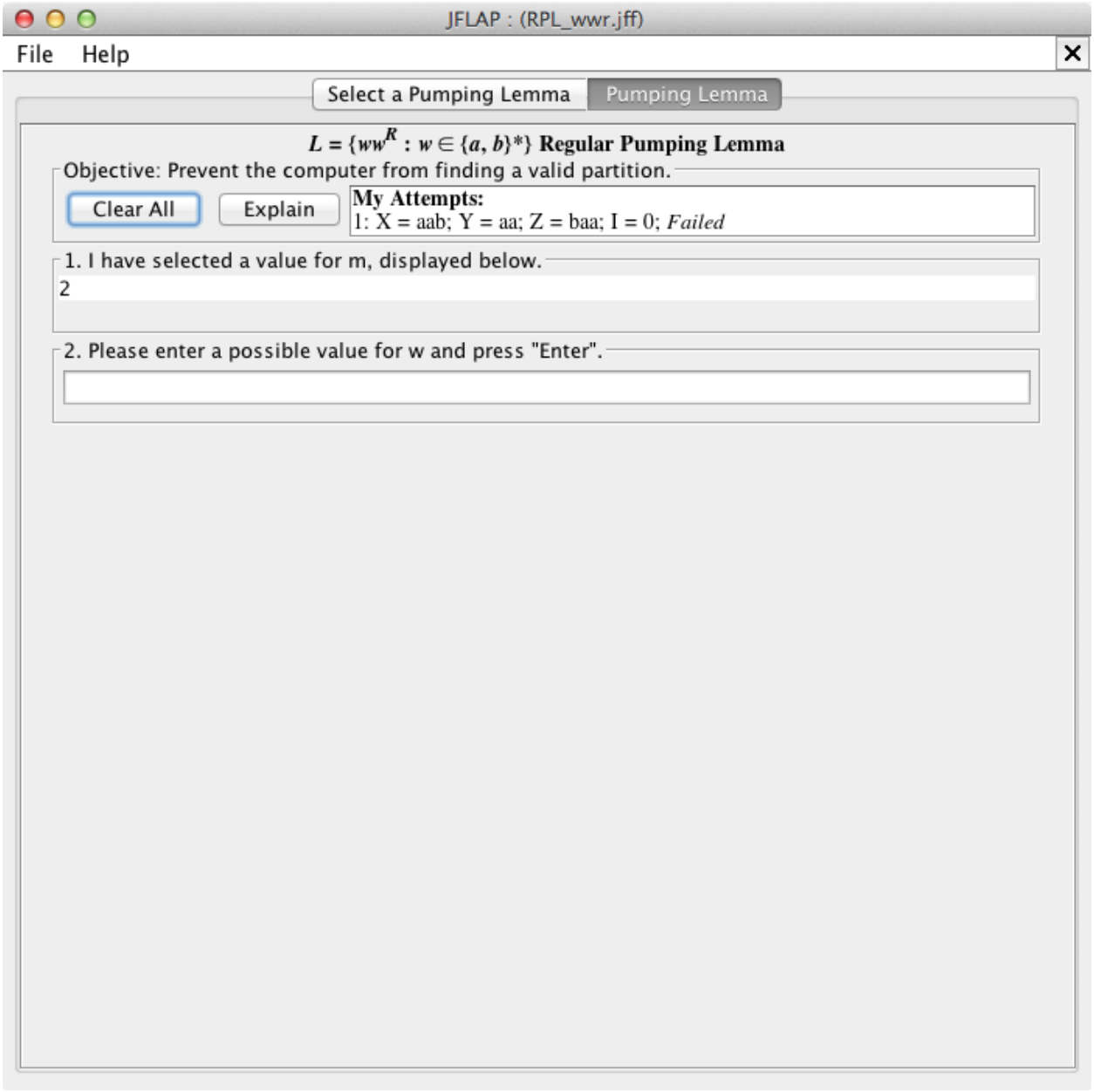
5. Step through the animation to see how the pumped string is constructed.




JFLAP : (RPL_wwr.jff)
File Help
Select a Pumping Lemma
Pumping Lemma
$L = \{ww^R : w \in \{a, b\}^*\}$ Regular Pumping Lemma
Objective: Prevent the computer from finding a valid partition.
Clear All
Explain
My Attempts:
1: X = aab; Y = aa; Z = baa; I = 0; Failed
1. I have selected a value for m, displayed below.
7
2. Please enter a possible value for w and press "Enter".
aabaabaa
3. I have decomposed w into the following...
X = aab; Y = aa; Z = baa
4. Please enter a possible value for i and press "Enter".
i: 0
pumped string: aabbaa
5. Animation
x y z
w = aab aa baa
aabbaa
$xy^0z = a^2b^2a^2$ = aabbaa is in the language. Please try again.
Step
Restart

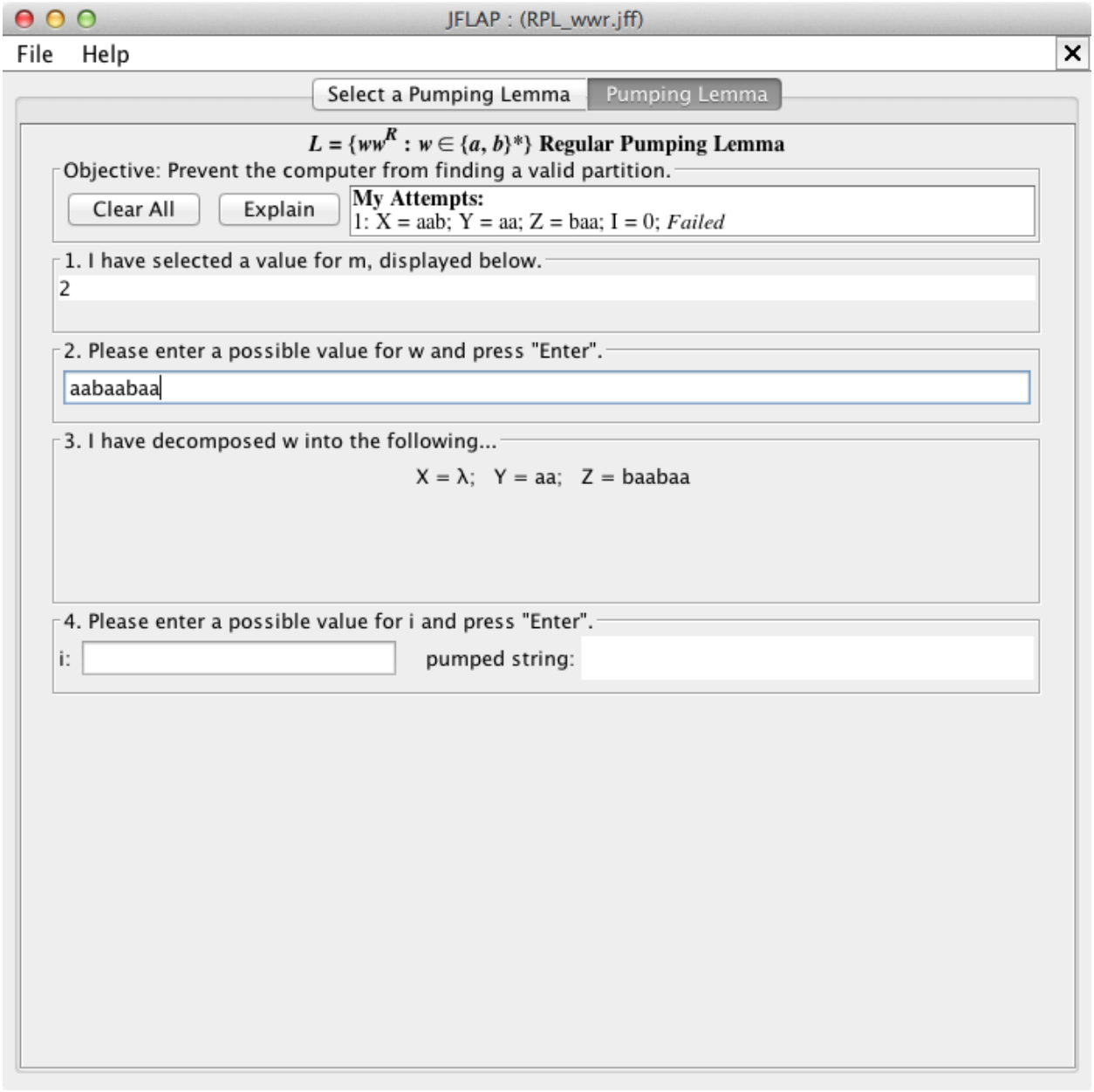
6. Now choose *Clear All* to try another string.





This time the computer has selected m = 2.

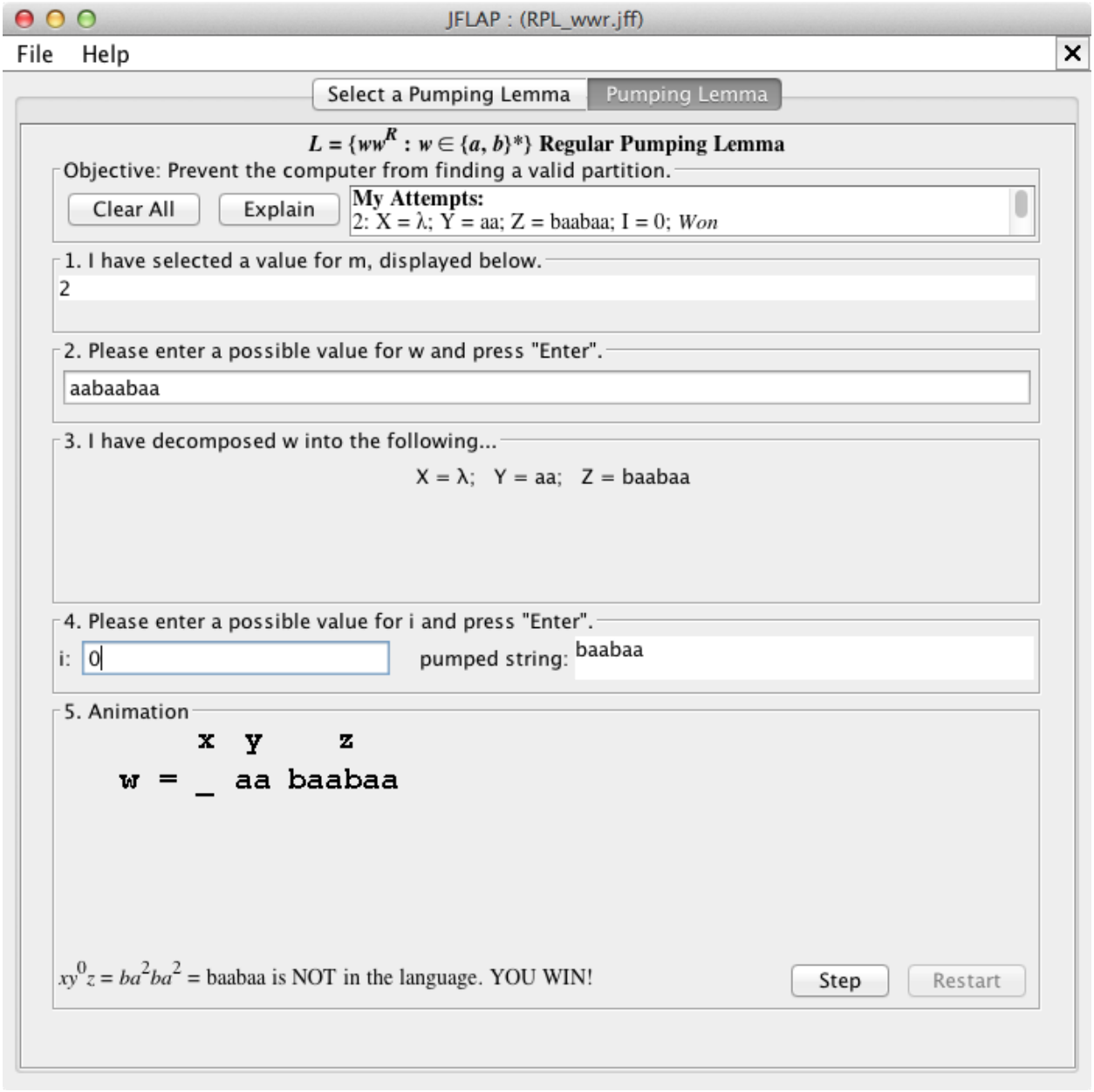
7. Consider the pumping lemma requirement that $|xy| \leq m$. Consider whether the previously constructed string, aabaabaa, would produce strings in language L when pumped. Try this string again.



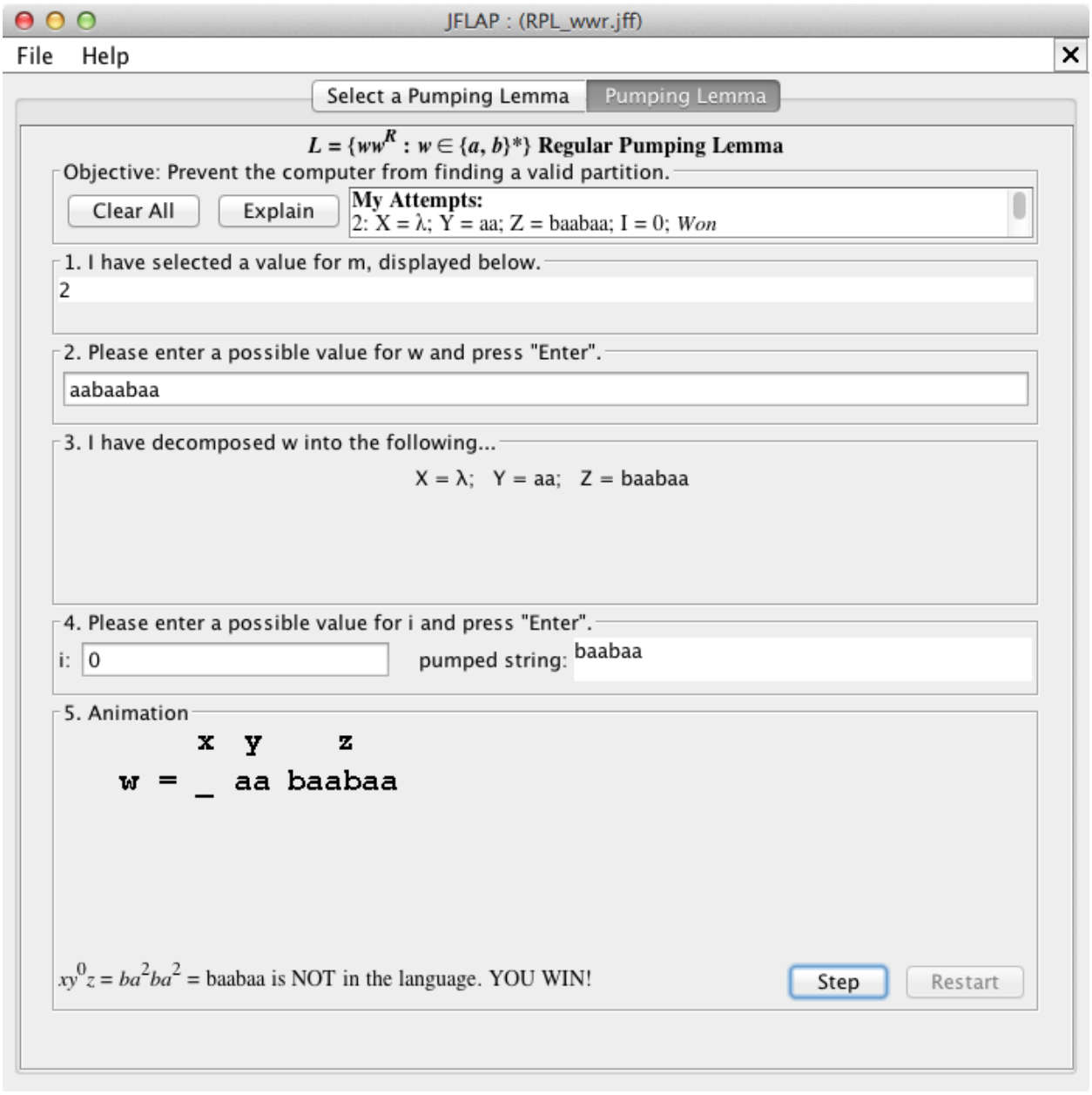
8. Consider which values for *i* would result in a string that is not in language L.

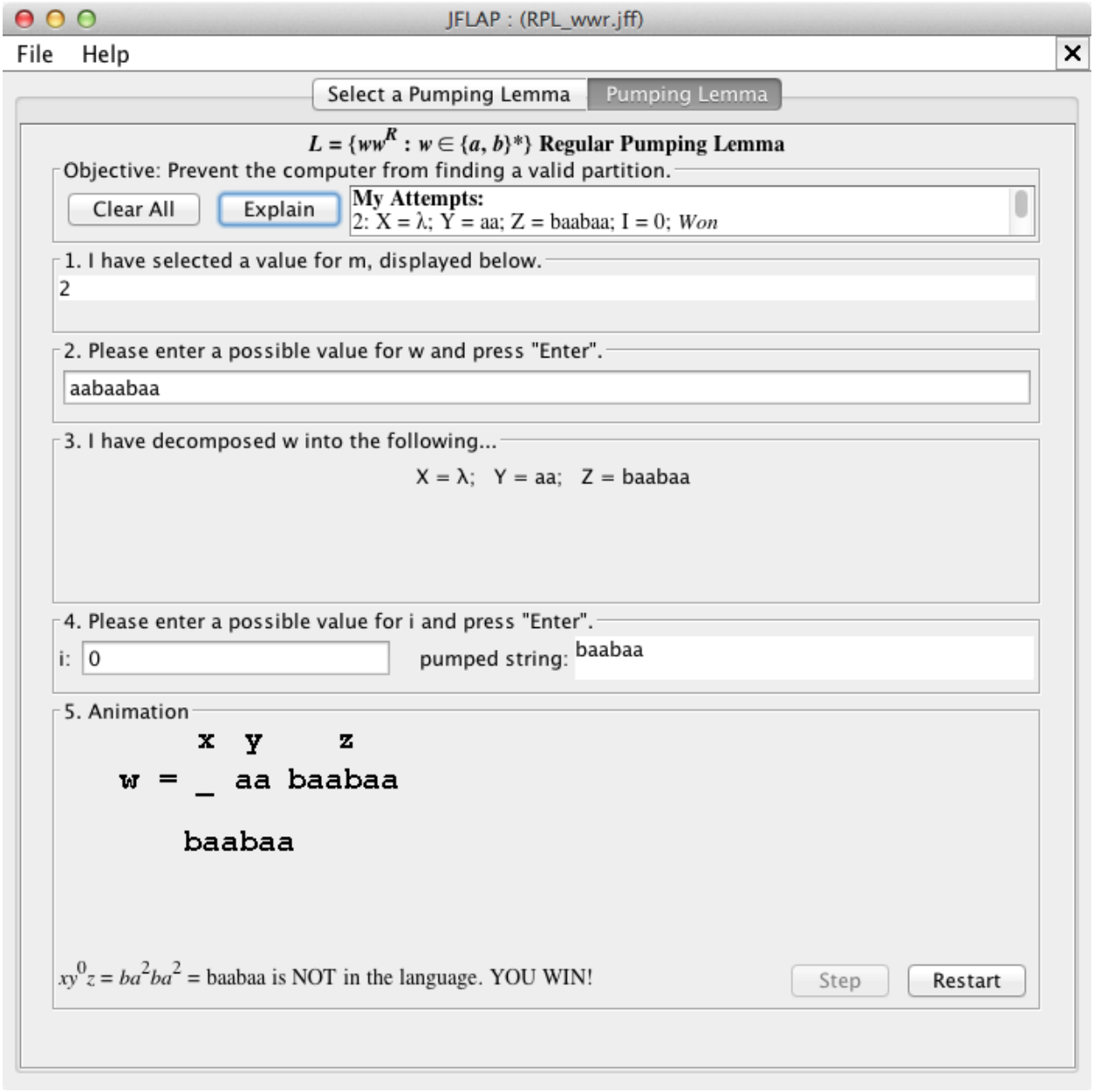
All values of $i \neq 1$ will work. That is, only in the case where $i = 1$ (the original string) is xy^iz □ L.

For this example, enter 0.

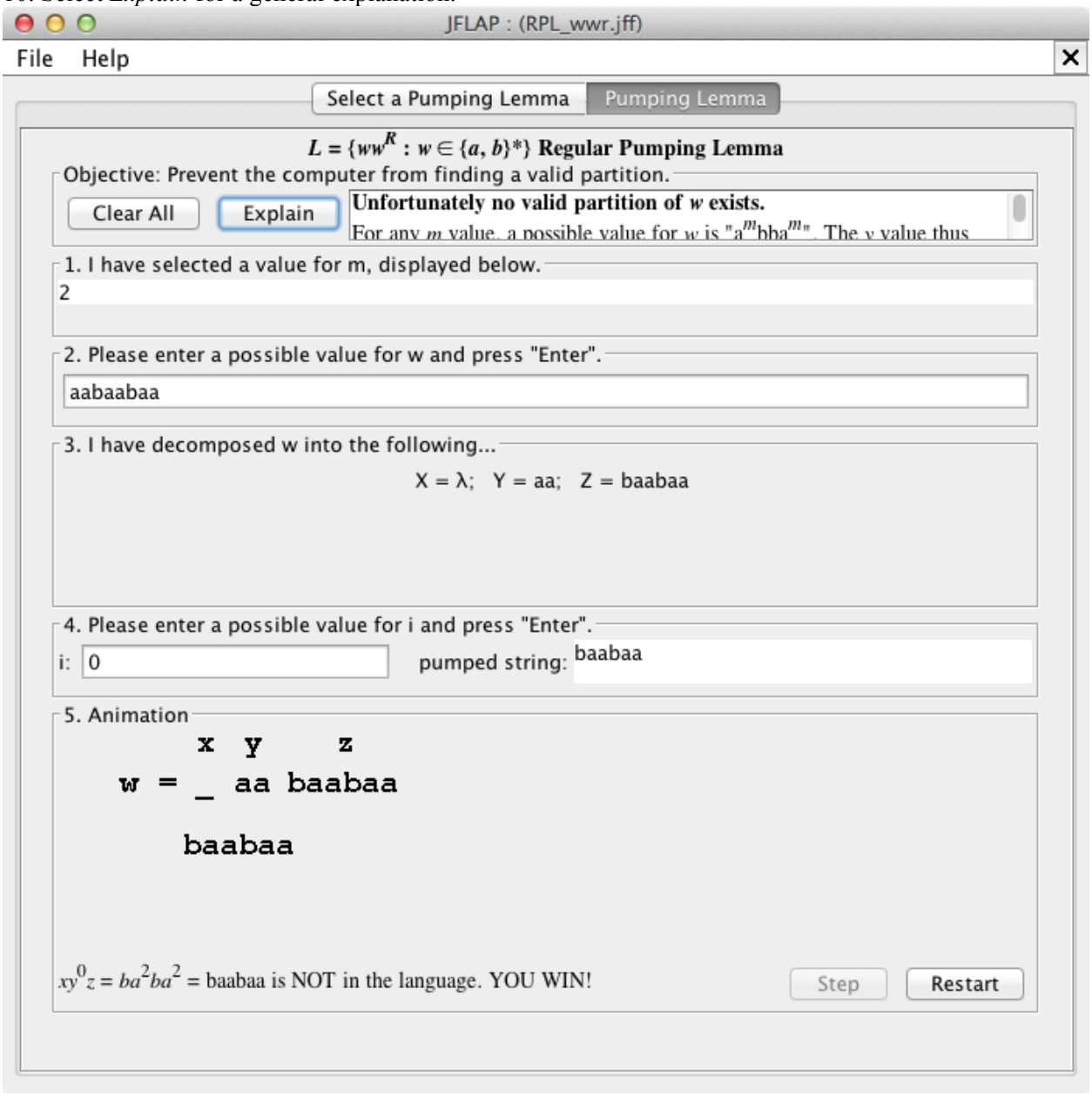

JFLAP : (RPL_wwr.jff)
File Help
Select a Pumping Lemma
Pumping Lemma
$L = \{ww^R : w \in \{a, b\}^*\}$ Regular Pumping Lemma
Objective: Prevent the computer from finding a valid partition.
Clear All
Explain
My Attempts:
2: X = λ; Y = aa; Z = baabaa; I = 0; Won
1. I have selected a value for m, displayed below.
2
2. Please enter a possible value for w and press "Enter".
aabaabaa
3. I have decomposed w into the following...
X = λ; Y = aa; Z = baabaa
4. Please enter a possible value for i and press "Enter".
i: 0
pumped string: baabaa
5. Animation
x y z
w = _ aa baabaa
$xy^0z = ba^2ba^2$ = baabaa is NOT in the language. YOU WIN!
Step
Restart

9. Select *Step* to see how the pumped string was constructed.




JFLAP : (RPL_wwr.jff)
File Help
Select a Pumping Lemma
Pumping Lemma
$L = \{ww^R : w \in \{a, b\}^*\}$ Regular Pumping Lemma
Objective: Prevent the computer from finding a valid partition.
Clear All
Explain
My Attempts:
2: X = λ; Y = aa; Z = baabaa; I = 0; Won
1. I have selected a value for m, displayed below.
2
2. Please enter a possible value for w and press "Enter".
aabaabaa
3. I have decomposed w into the following...
X = λ; Y = aa; Z = baabaa
4. Please enter a possible value for i and press "Enter".
i: 0
pumped string: baabaa
5. Animation
x y z
w = _ aa baabaa
baabaa
$xy^0z = ba^2ba^2$ = baabaa is NOT in the language. YOU WIN!
Step
Restart

10. Select *Explain* for a general explanation.



Note that you may need to scroll to read the full explanation, which is reproduced here.

> **Unfortunately no valid partition of *w* exists.**
> For any *m* value, a possible value for *w* is "$a^m bba^m$". The *y* value thus would be a multiple of "a" in 'w' and not in 'w^R'. If *i* = 0, then the total string becomes at most "$a^{m-1} bba^m$", which is not in the language. Thus, the language is not regular.
>
> **My Attempts:**
> 2: X = λ; Y = aa; Z = baabaa; I = 0; *Won*
> 1: X = aab; Y = aa; Z = baa; I = 0; *Failed*